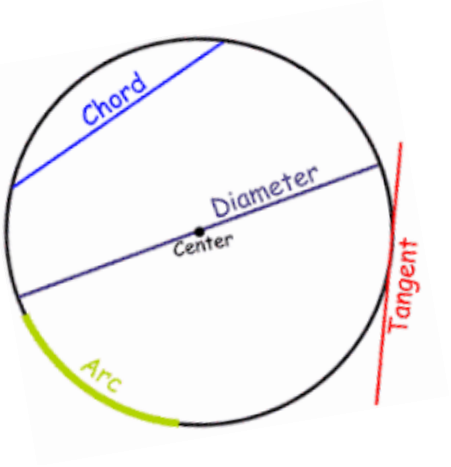
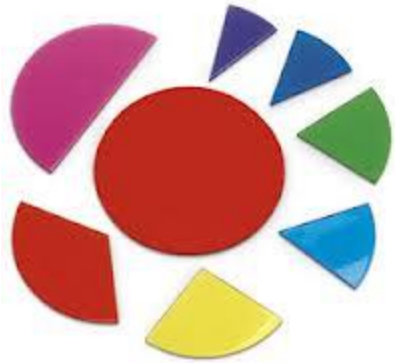
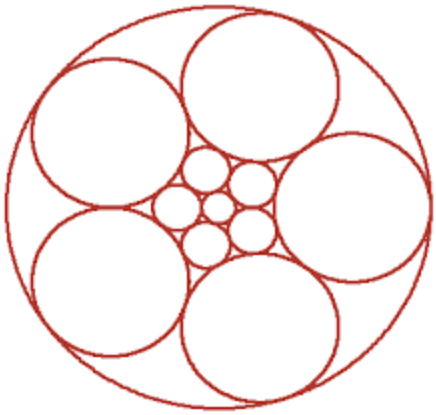
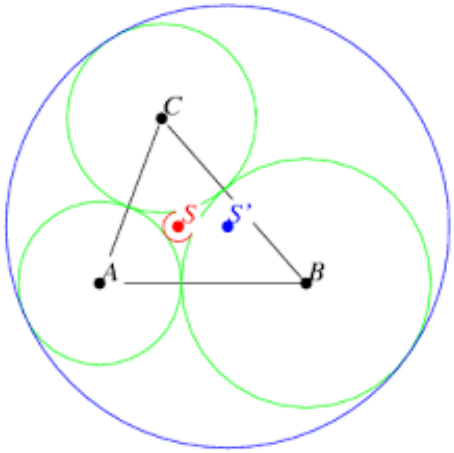
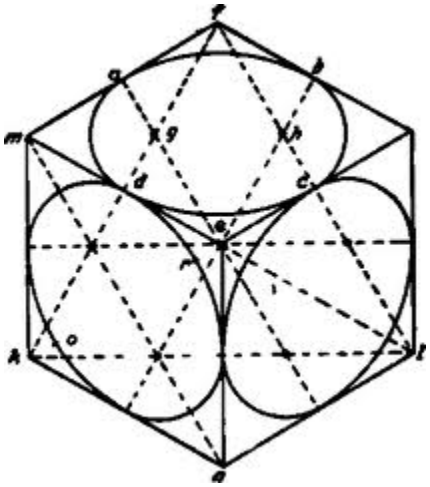


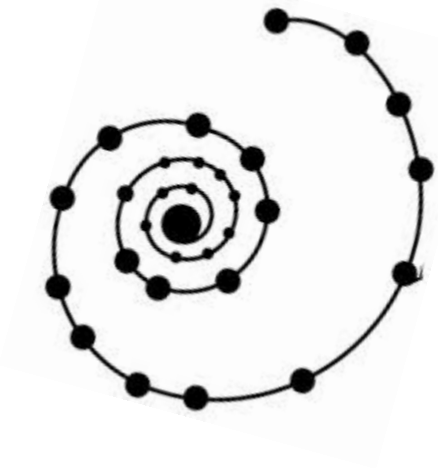




Geometry
Unit 8
CIRCLES







Content adapted from Department of Education State Task at www.doe.k12.ga.us.

Learning Task: Proving Circles Are Similar

Materials needed: Compass, ruler, and calculator.

1.

a. Using your compass, draw a circle with any radius.

b. Label the center of the circle and select any point on the circle. Connect the two points. This segment is known as the radius.

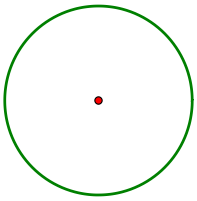
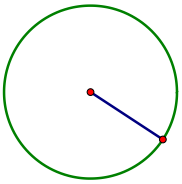
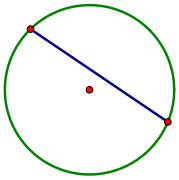
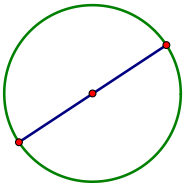
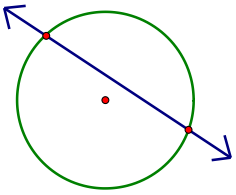
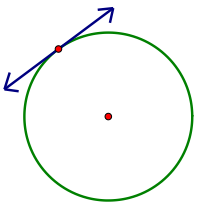
c. Measure the length of your radius. Use the formula $C = 2\pi r$ to calculate the circumference of your circle. Leave your answers in terms of π.

d. Write your answer from part (c) as a fraction over the diameter of the circle. Simplify. (Hint: Diameter = 2 times the radius)

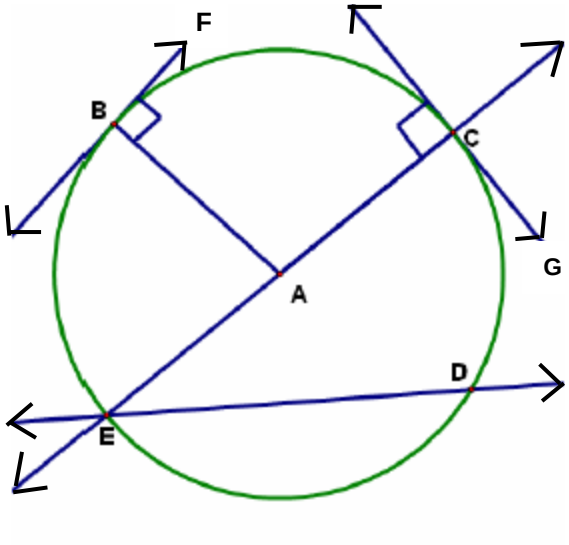
e. Compare your answer with your group members. What do you notice?

f. What does this mean in regards to all circles?

Learning Task: Lines and Line Segments of a Circle Graphic Organizer

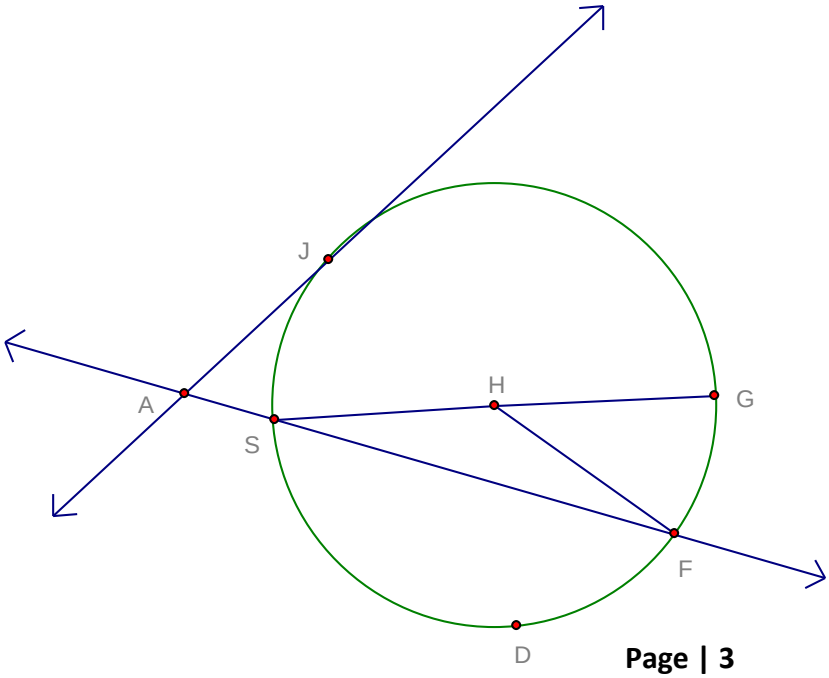
Vocabulary Word	Definition	Drawing
Circle		
Center		
Radius		
Chord		
Diameter		
Secant		
Tangent		

Example: Using the diagram to the right, give an example of each of the following. Be sure to use proper notation.



1. Center
2. Chord (other than the diameter)
3. Diameter
4. Radius
5. Tangent
6. Point of Tangency
7. Secant

Skills Practice: Using the diagram, give an example of each of the following. Be sure to use proper notation.

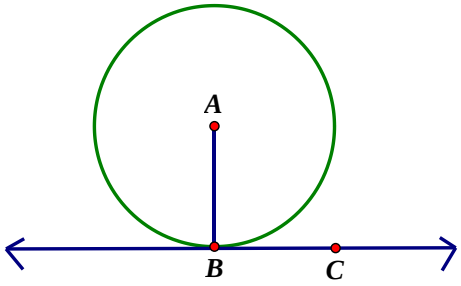


1. Circle
2. Secant
3. Tangent
4. Radius
5. Diameter
6. Point of Tangency
7. Chord

Learning Task: Tangents

$\overleftrightarrow{BC}$ is tangent to $\odot A$ at point B.

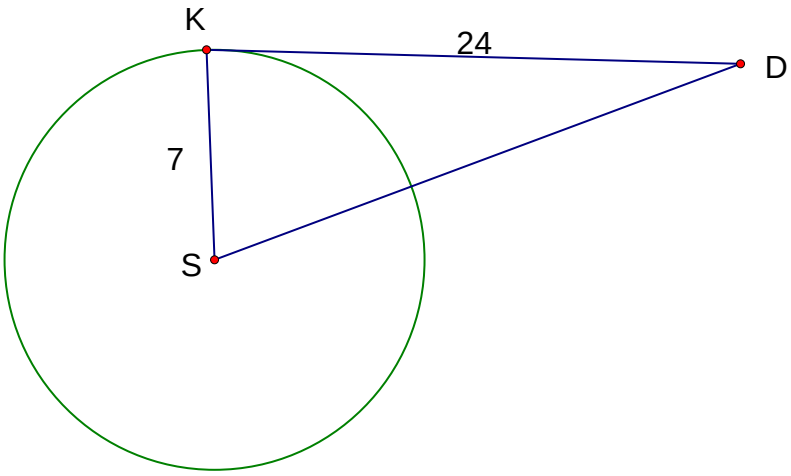
What type of segment is $\overline{AB}$ **?**



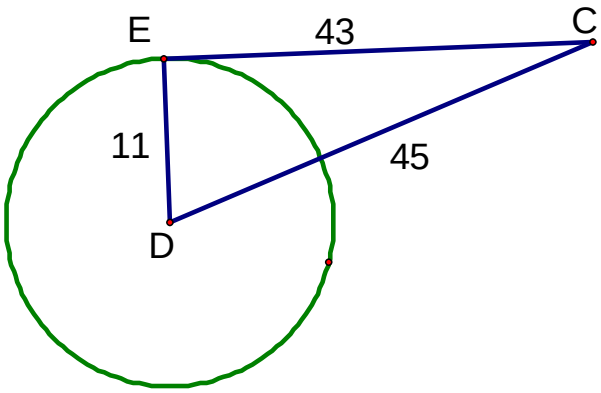
Find $m\angle ABC$ = _____

What is the relationship between the radius and the tangent line that intersect at the point of tangency?

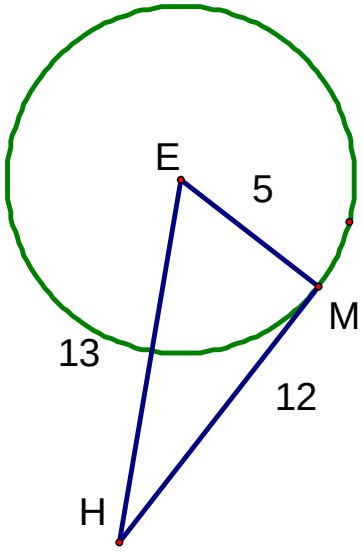
1. If $\overline{KD}$ is tangent to circle S, find the length of $\overline{SD}$.



2. Determine if $\overline{EC}$ is tangent to circle D. Explain your answer.

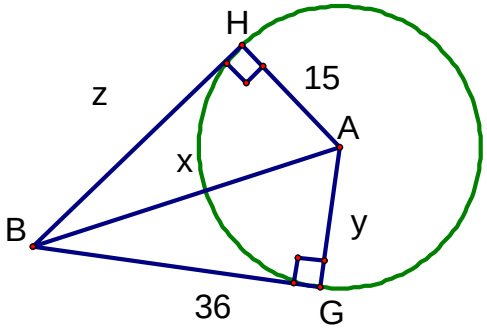


3. Is segment MH tangent to circle E? Justify your answer.



How are the two tangent segments to a circle that intersect at a point outside the circle related?

Find the values of x, y, and z in the diagram.



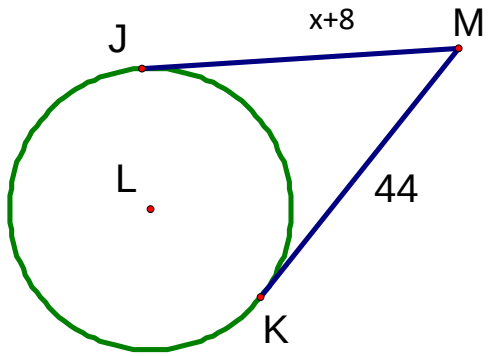
What do you notice about the lengths of BH and BG?

Since BH and BG are tangent to the circle, what can you conclude about the relationship of two tangents from the circle intersecting at one point outside the circle?

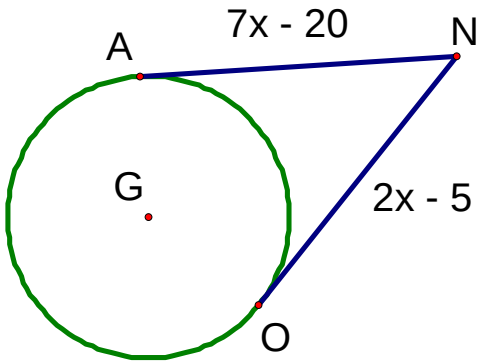
THEOREM: Two tangent segments are _____ if they intersect the same point outside the circle.

Guided Practice: Apply theorems relating to tangents of a circle

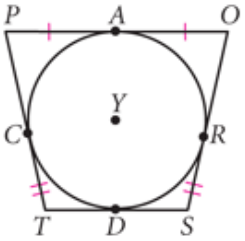
1. $\overline{JM}$ and $\overline{MK}$ are tangent to circle L. Find the value of x.



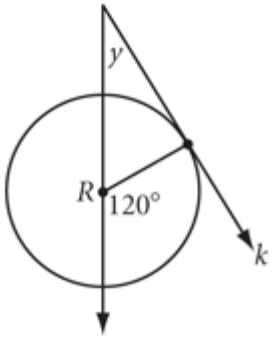
2. $\overline{NA}$ and $\overline{NO}$ are tangent to circle G. Find the value of x.



3. Quadrilateral POST is circumscribed about circle Y. OR = 13 in. and ST = 12 in. Find the perimeter of POST.

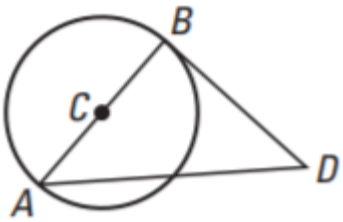


4. Ray k is tangent to circle R. What is the value of y?



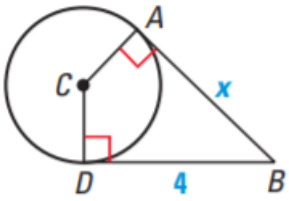
Skills Practice

1. In the diagram below, AB = BD = 5 and AD = 7. Is $\overline{BD}$ tangent to $\odot C$? Explain.

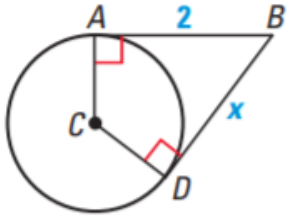


2. $\overline{AB}$ is tangent to $\odot C$ at A and $\overline{DB}$ is tangent to $\odot C$ at D. Find the value of x.

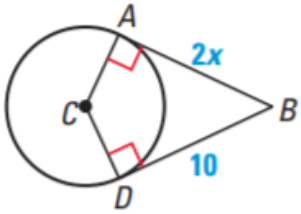
a.



b.

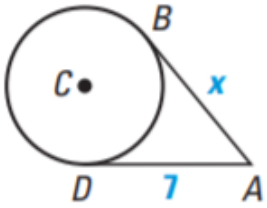


c.

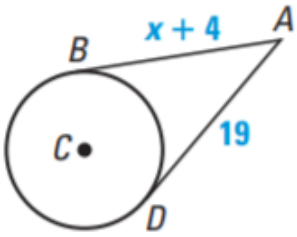


3. $\overline{AB}$ and $\overline{AD}$ are tangent to $\odot C$. Find the value of x.

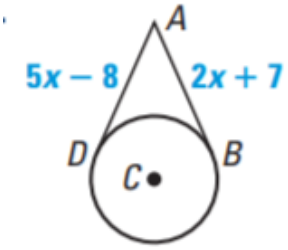
a.



b.

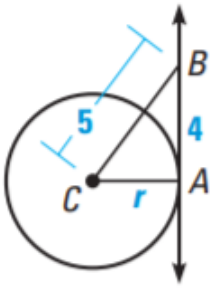


c.

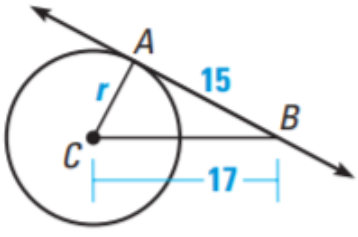


4. $\overleftrightarrow{AB}$ is tangent to $\odot C$. Find the value of r.

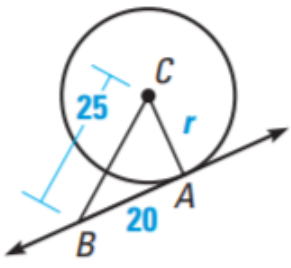
a.



b.

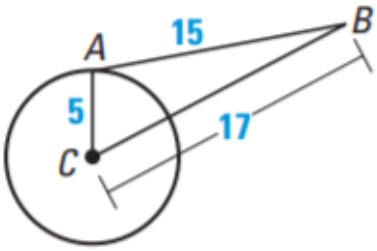


c.

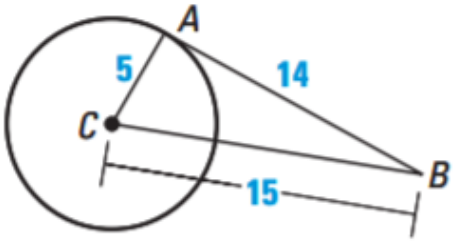


5. Tell whether $\overline{AB}$ is tangent to $\odot C$. Explain your reasoning.

a.



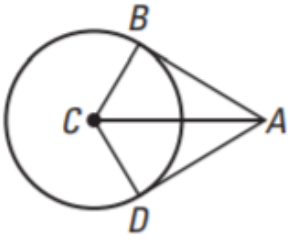
b.



6. $\overline{AB}$ and $\overline{AD}$ are tangent to $\odot C$.

a. Name all congruent segments

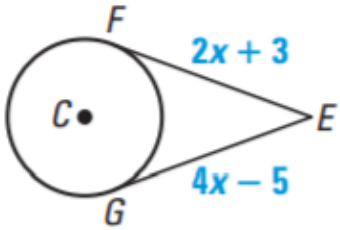
b. Name all congruent angles.



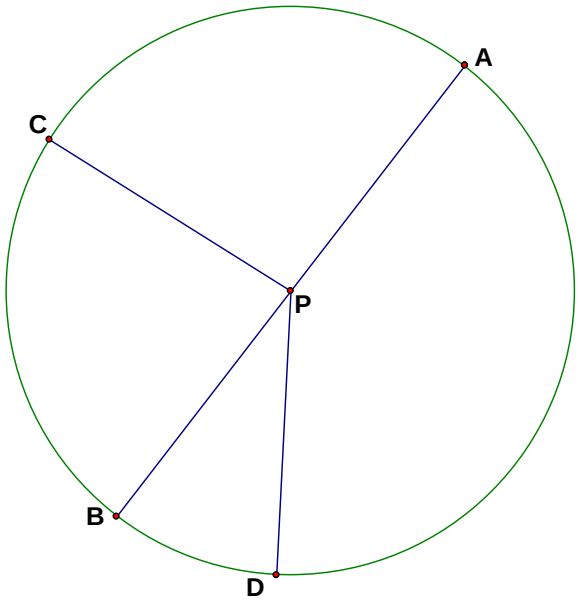
c. Name two congruent triangles.

7. MULTIPLE CHOICE: In the diagram below, $\overline{EF}$ and $\overline{EG}$ are tangent to $\odot C$. What is the value of x?

A. -4
B. -1
C. 1
D. 4



Learning Task: Central Angles and Arcs of a Circle Graphic Organizer

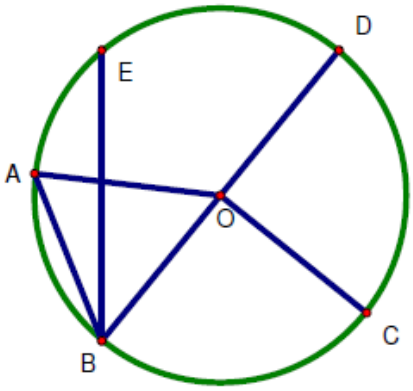


Vocabulary Word	**Definition**	**Example** (using correct notation)
Central Angle		
Arc		
Semicircle		
Minor Arc		
Major Arc		
Congruent Arcs		

Central Angle = _____ measure

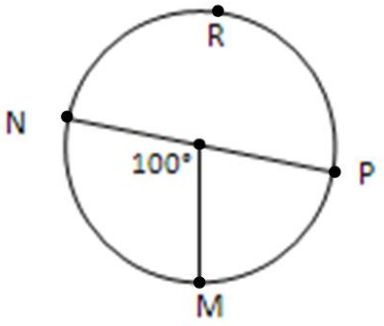
Example: Use the figure to the right to identify and name the following:

1. Two different central angles
2. A minor arc
3. A major arc
4. A diameter
5. A semicircle



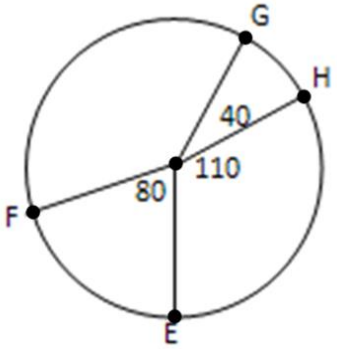
Guided Practice: In problems 1-4, $\overline{NP}$ is a diameter. Find the indicated measures.

1. $\widehat{MN}$
2. $\widehat{MPN}$
3. $\widehat{PMN}$
4. $\widehat{PM}$



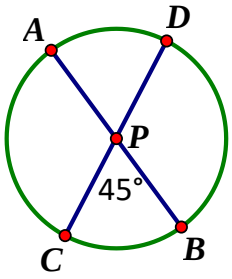
Use the figure to the right to find the measure of the indicate arcs.

5. $\widehat{GE}$
6. $\widehat{GEF}$
7. $\widehat{GF}$
8. $\widehat{FHE}$

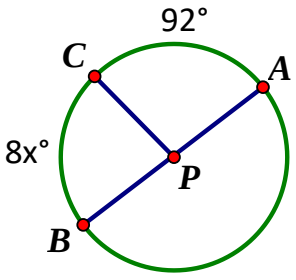


In problems 9-11, use Circle P to find the value of x and then find the arc measures. Pictures are not drawn to scale.

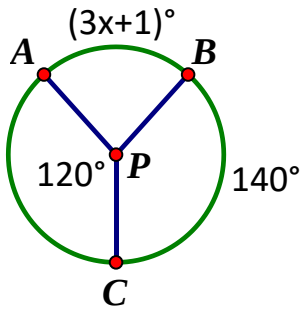
9.



10.

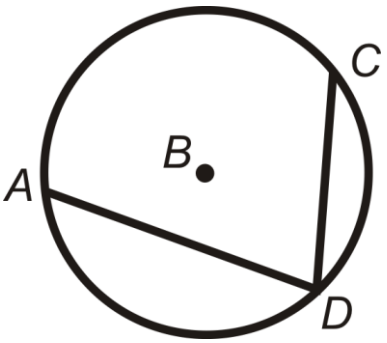


11.



Inscribed Angles

1. $\angle ADC$ is called an inscribed angle. Explain why.



2. AC is called the intercepted arc of $\angle ADC$. Explain why.

3. Given the following measures, complete the theorem below.

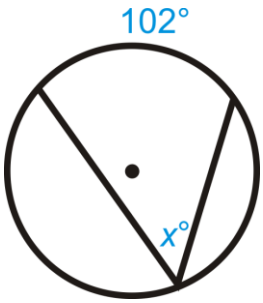
$m\angle ADC = 35°$ $mAC = 70°$

Inscribed Angle Theorem
The measure of an inscribed angle is _____ _____ the measure of its intercepted arc.

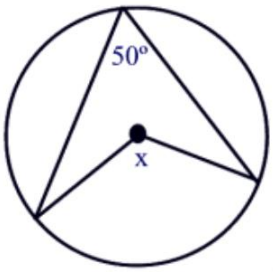
In other words, the intercepted arc is _____ the measure of the inscribed angle.

Guided Examples: Find the measure of the missing variable.

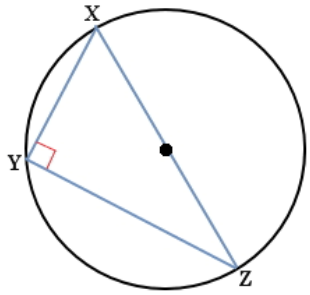
1.



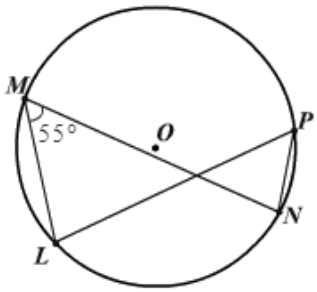
2.



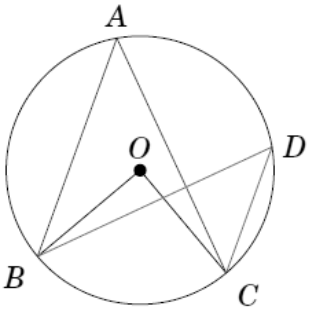
3. Find the measure of arc XZ



4. Find the measure of angle LPN.



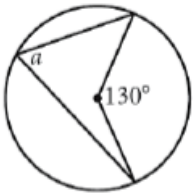
Theorem
If two inscribed angles of a circle intercept the same arc, then the angles are _____.



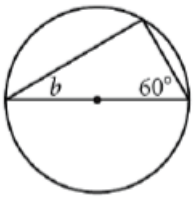
Skills Practice

Use the theorems to solve each question.

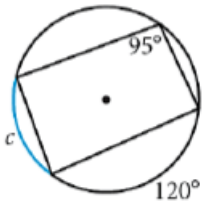
1. a = _____



2. b= _____

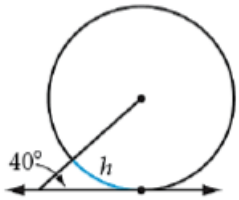


3. c = _____



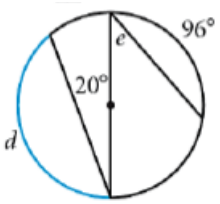
4. h = _____

Hint: draw a radius



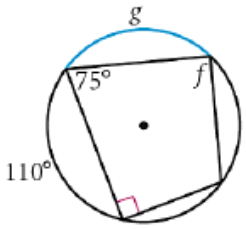
5. d = _____

e = _____



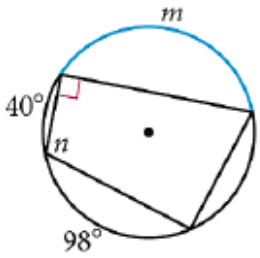
6. f = _____

g = _____

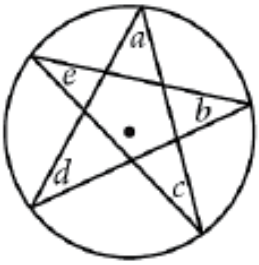


6. m = _____

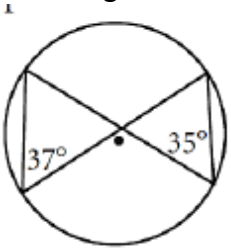
n = _____



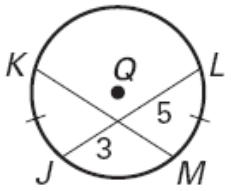
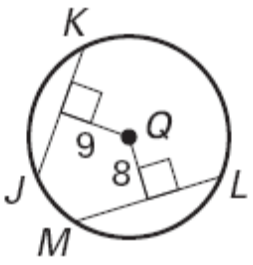
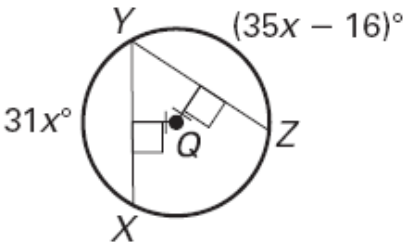
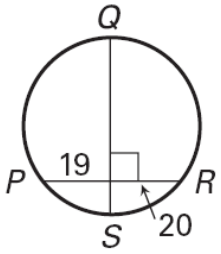
8. What is the sum of a, b, c, d, and e?



10. What is wrong with this picture?



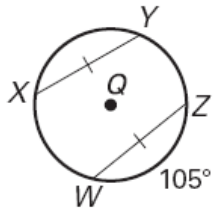
Learning Task: Theorems with Chords and Arcs

Theorem	Example
If two chords are congruent then their arcs are congruent	**Find the value of KM.** 
Two chords are congruent if they are equidistant from the center of the circle	**Are $\overline{JK}$ and $\overline{ML}$ congruent?** 
Two chords are congruent if and only if they are equidistant from the center of the circle.	**Find the measure of YX.** 
To be a diameter the chord must be a perpendicular bisector of another chord.	**Is $\overline{QS}$ a diameter? Why or why not?** 
Pythagorean Theorem.	**A chord in a circle is 18 cm long and is 5 cm from the center of the circle. What is the length of the radius of the circle?**

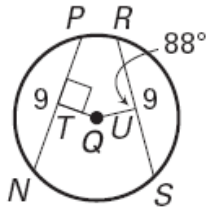
Skills Practice

Answer the following problems using the theorems from the previous page.

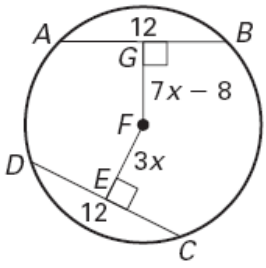
1. Find the measure of arc YZ if the measure of arc XW = 95°



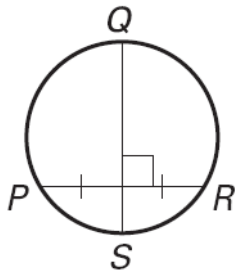
2. Are segments TQ and UQ congruent?



3. Find the measure of GF.



4. Is segment QS a diameter? Explain your reasoning.

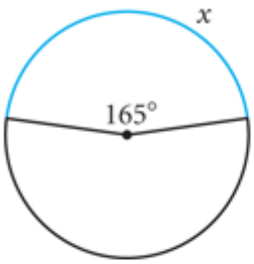


5. The chord of a circle is 15 inches and it is drawn 4 inches from the center of the circle. What is the length of the radius of the circle?

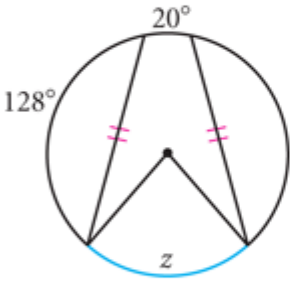
Additional Skills Practice

Solve for the missing variables.

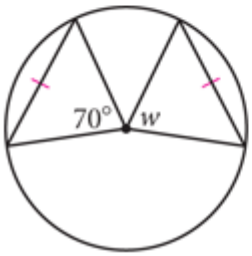
1. $x =$?



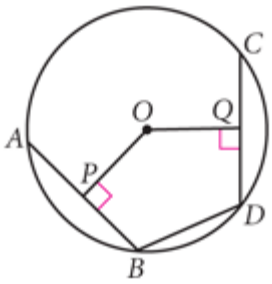
2. $z =$?



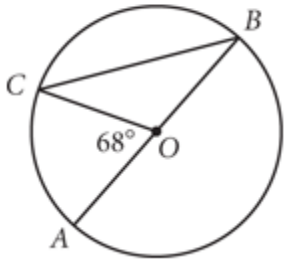
3. $w =$?



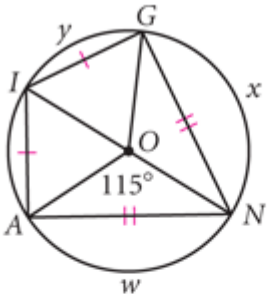
4. $AB = CD$
$PO = 8$ cm
$OQ =$?



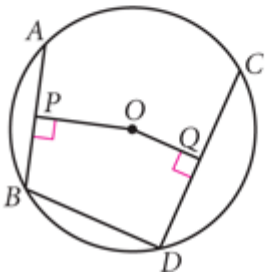
5. $\overline{AB}$ is a diameter.
Find $m\overset{\frown}{AC}$ and $m\angle B$.



6. *GIAN* is a kite. Find w, x, and y.

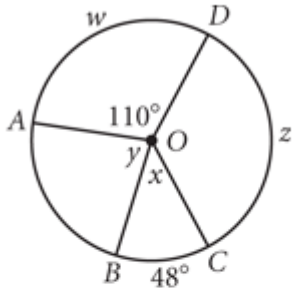


7. $AB = 6$ cm
$OP = 4$ cm
$CD = 8$ cm
$OQ = 3$ cm
$BD = 6$ cm
What is the perimeter of *OPBDQ*?



8. $m\overset{\frown}{AC} = 130^\circ$

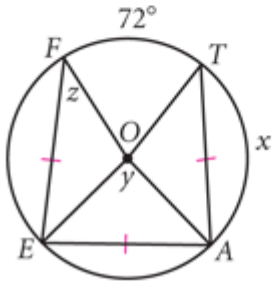
Find w, x, y, and z.



9. $x =$?

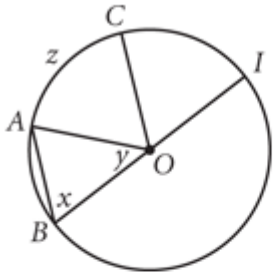
$y =$?

$z =$?

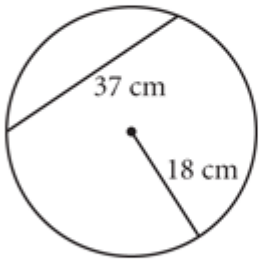


10. $AB \| CO, mCI = 66^\circ$

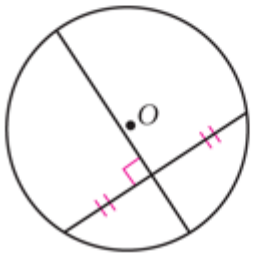
Find x, y, and z.



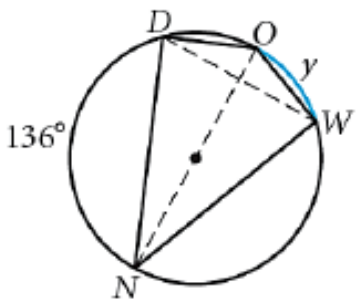
11. What's wrong with this picture?



12. What's wrong with this picture?



13. y = _____



Warm-up

Solve the following equations for *x*.

1. $46 = \frac{1}{2}(18x + 2)$

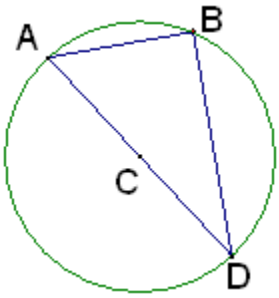
2. $14x + 3 = \frac{1}{2}(90)$

Learning Task : Central Angles and Arcs of a Circle Graphic Organizer

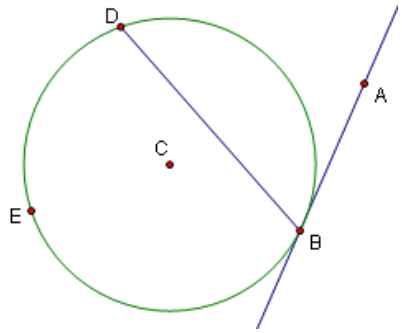
Location of the Vertex	Picture	Theorem
Inside the Circle • At the center • Not at the center		
Outside the Circle		
On the circle		

Guided Practice:

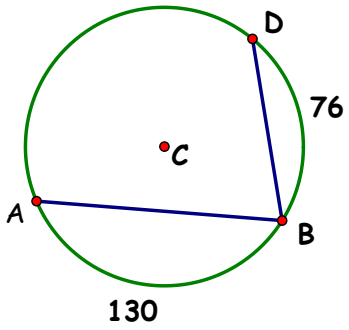
1. Find the m∠ABD, the inscribed angle of ⊙C.



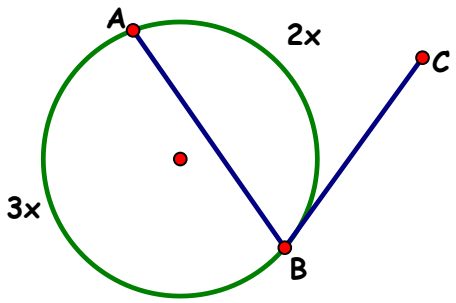
2. Find the m∠ABD, the inscribed angle of ⊙C, if $mBED = 300^{\circ}$.



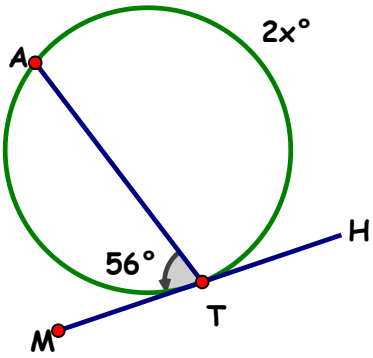
3. Find the m$\angle ABD$, the inscribed angle of ⊙C.



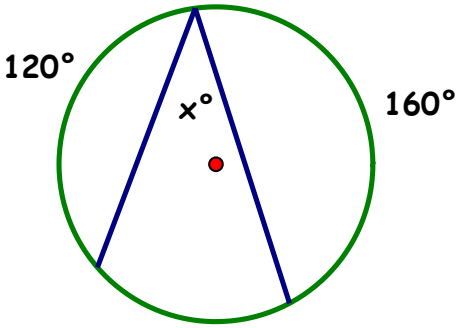
4. Find the measure of $\angle ABC$.



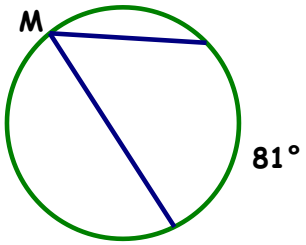
5. Solve for x.



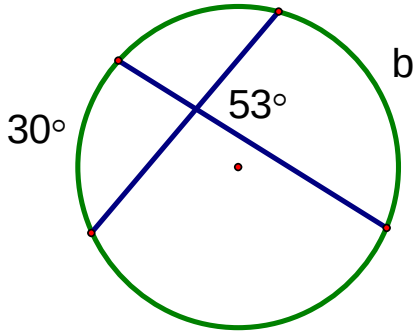
6. Solve for x.



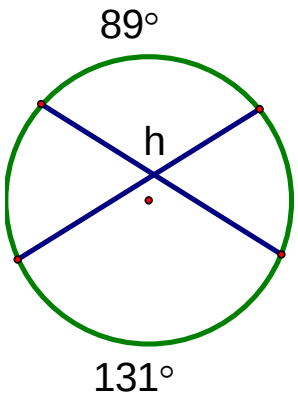
7. Find the measure of angle M.



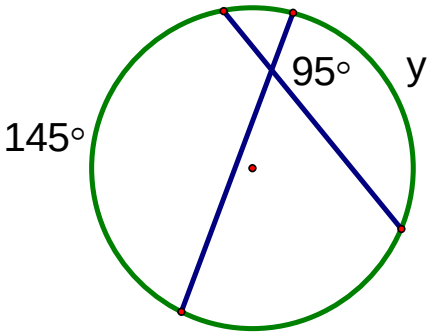
8. Find $m(arcB)$.



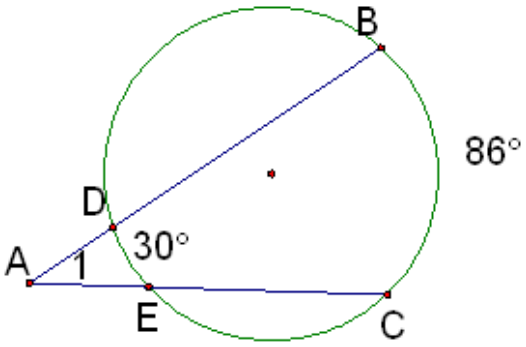
9. Find $m\angle h$.



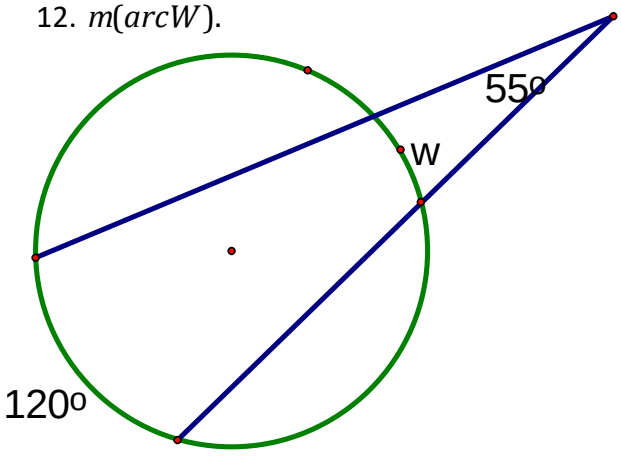
10. Find $m(arcY)$.



11. Find $m\angle 1$



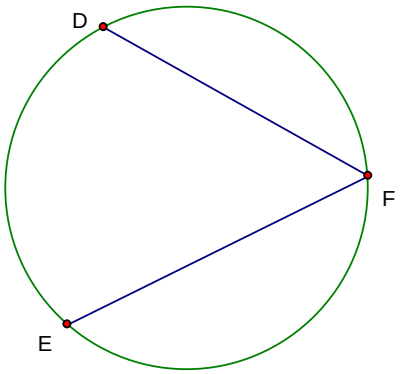
12. $m(arcW)$.



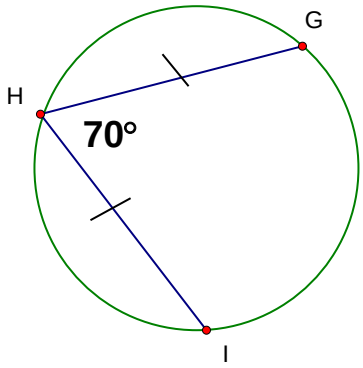
Skills Practice

Find the indicate arc/angle or missing variable.

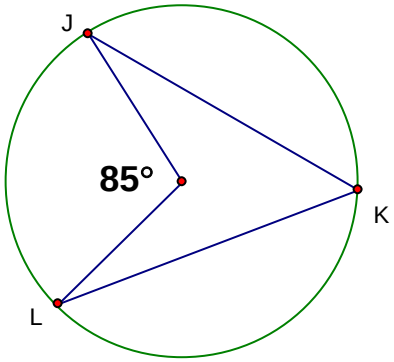
1. $mDE = 124°$
   Find $m\angle DFE$



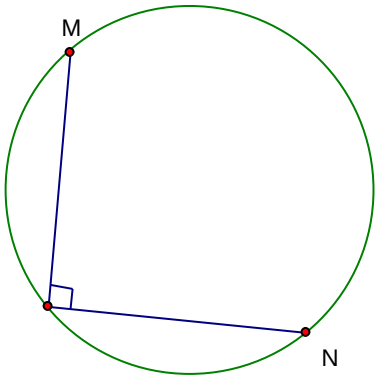
2. Find mGH.



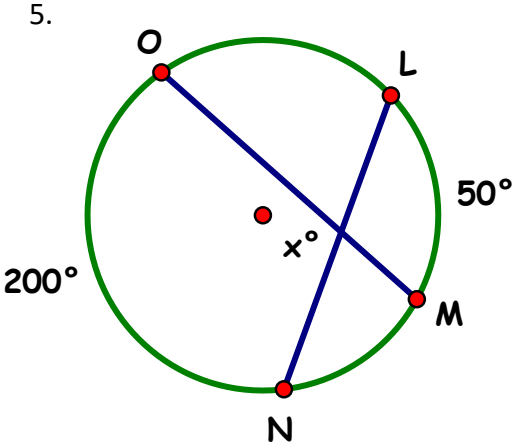
3. Find $m\angle JKL$



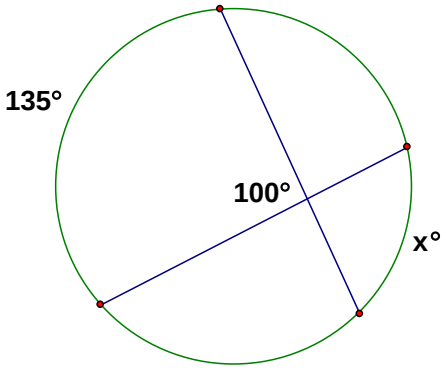
4. $mMN = (4x + 8)°$
   Solve for x.



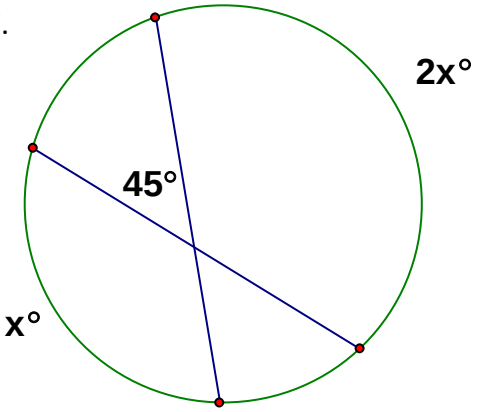
5.



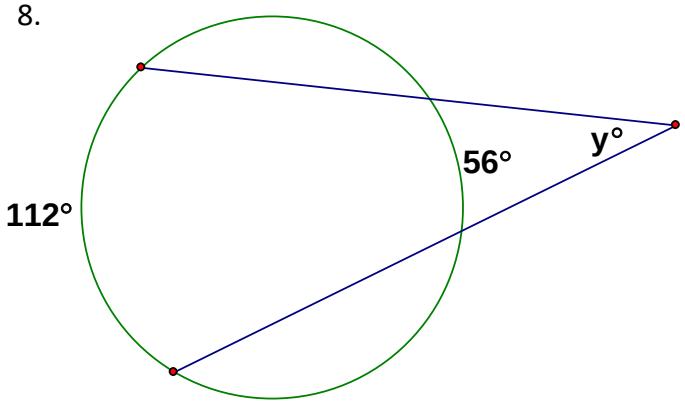
6.



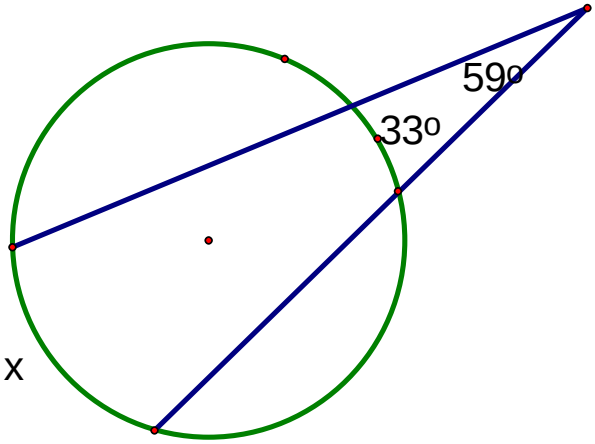
7.



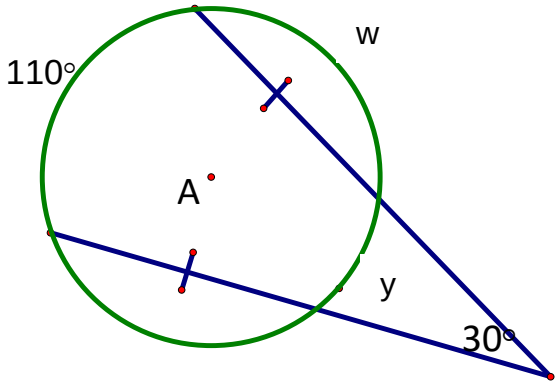
8.



9.



10.



Warm-Up

In problems 1-4, solve for *x.*

1. $12^2 + x^2 = 20^2$

2. $14(2x+1) = 10(3x)$

3. $21(6x+2) = 30(4x+2)$

4. $6^2 = 4(4+x)$

5. **Recall the following definitions:**

   A _____ _____ is a line that intersects a circle in exactly one point.
   A _____ _____ intersects a circle in two points.

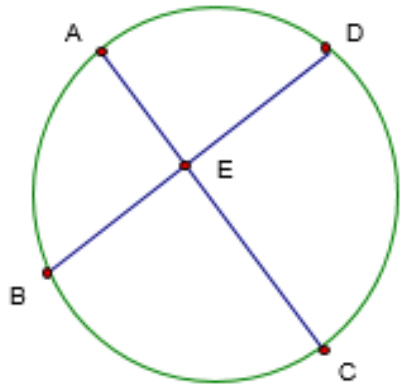
6. Is it possible for a line to intersect a circle in 3 points? 4 points? Explain why or why not.

7. When a secant line intersects a circle in two points, it creates a chord. As you have already learned, **a chord is a segment whose endpoints lie on the circle. How does a chord differ from a secant line?**

Learning Task: Segment Lengths of a Circle

Case 1 - Two Chords Intersecting Inside a Circle

Example 1: In the circle below, AE = 4 EC = 12 BE = 8 ED = 6

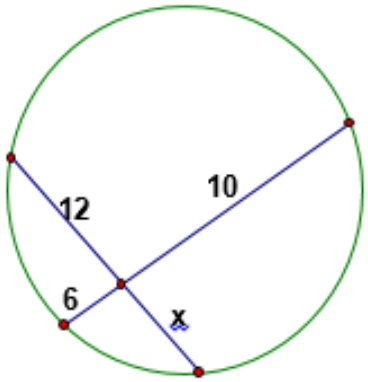


Calculate:

AE ● EC = _____ BE ● ED = _____

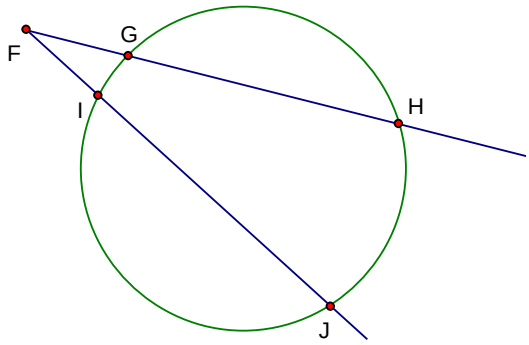
What do you notice about the products of the lengths?

Example 2: Find the value of x.



Case 2 - Two Secants Intersecting Outside the Circle

Example 1:



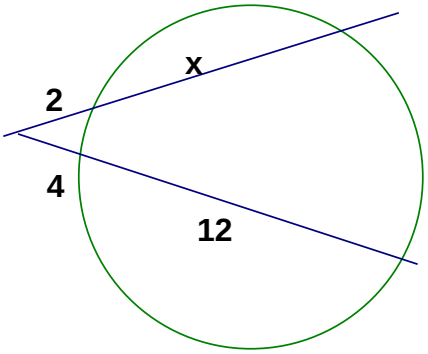
Given: FG = 5 FI = 4 GH = 3 IJ = 6

Calculate:

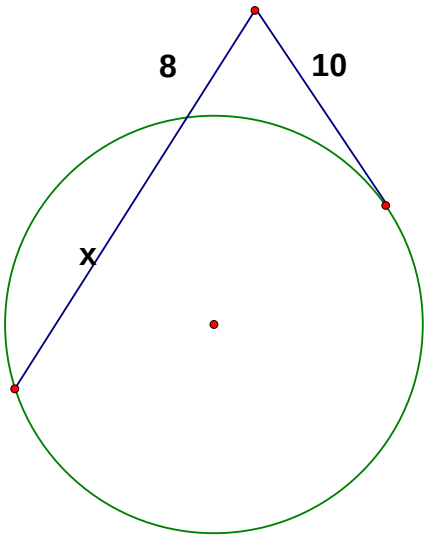
FG ● FH = _____ FI ● FJ = _____

What do you notice about the lengths being multiplied?

Example 2: Find the value of x.

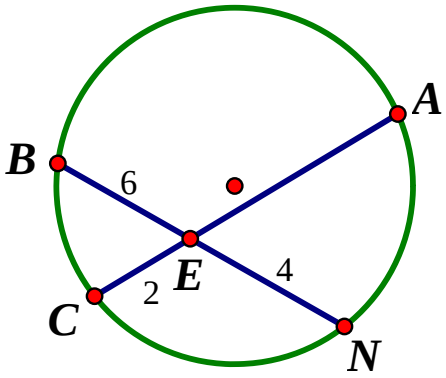


Example 3: Apply your knowledge of two secants intersecting outside the circle to the following:

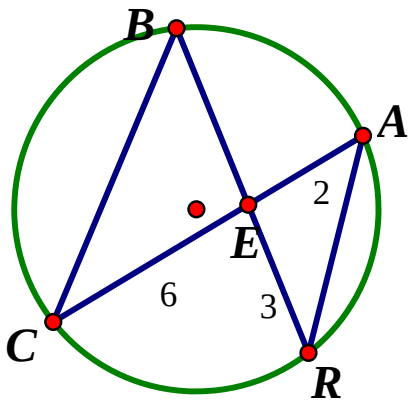


Guided Practice: Find the missing indicated segment in each of the following examples.

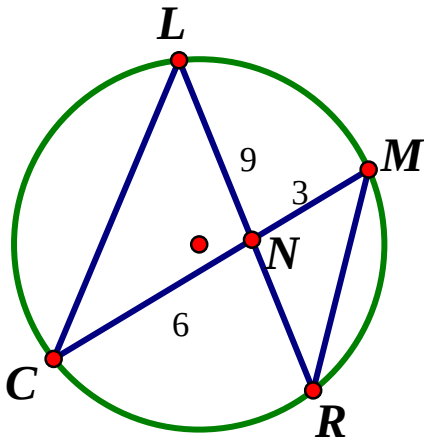
1. Find AE.



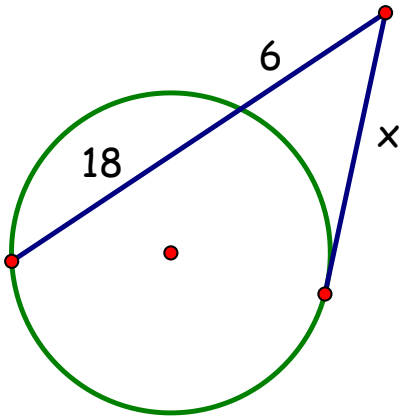
2. Find BE.



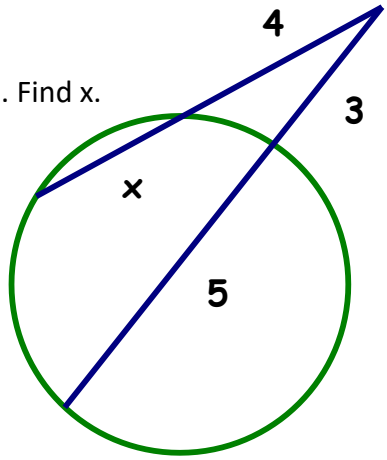
3. Find NR.



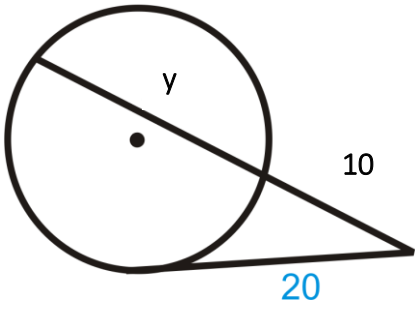
4. Find x.



5. Find x.

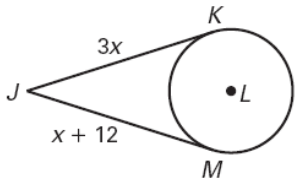
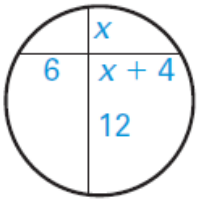
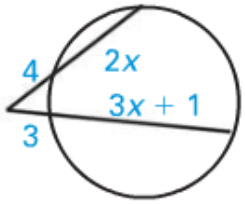
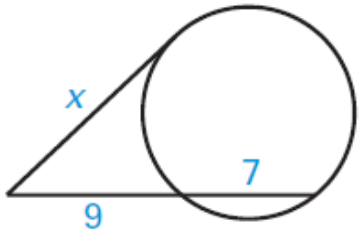
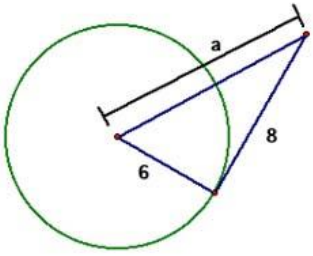


6.



The Segment Theorems Graphic Organizer

Let's summarize the theorems relating to tangents, chords, and secants. Use the information from the previous task to complete the graphic organizer.

Picture	Type	Theorem	Example
			Find **JK** 
			Solve for **x.** 
			Find the value of **x.** 
			Find the value of **x.** 
			Find the value of a. 

Skills Practice

Sketch a picture for each problem, choose a theorem, set up an equation, and then solve.

1. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. *AE= 2, CE =4*, and *ED 3*. Find *EB*.

2. A diameter of a circle is perpendicular to a chord whose length is 12 inches. If the length of the shorter segment of the diameter is 4 inches, what is the length of the longer segment of the diameter?

3. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. *AE= 5,* CE *=10, EB = x,* and *ED = x-4*.
Find *EB* and *ED*.

4. Two secant segments are drawn to a circle from a point outside the circle. The external segment of the first secant segment is 8 centimeters and its internal segment is 6 centimeters.
If the entire length of the second secant segment is 28 centimeters, what is the length of its external segment?

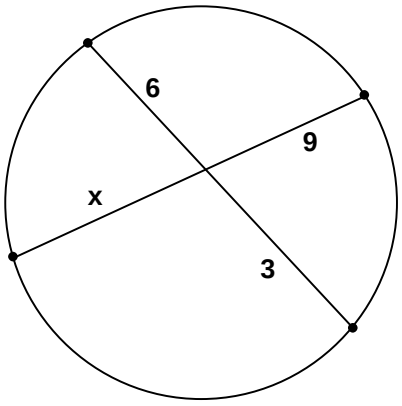
5. A tangent segment and a secant segment are drawn to a circle from a point outside the circle.
The length of the tangent segment is 15 inches. The external segment of the secant segment measures 5 inches. What is the measure of the internal secant segment?

6. The diameter of a circle is 19 inches. If the diameter is extended 5 inches beyond the circle to point *C*, how long is the tangent segment from point *C* to the circle?

<u>Skills Practice</u> (continued)

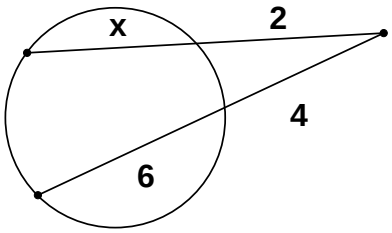
Find the value of the missing variable.

1.



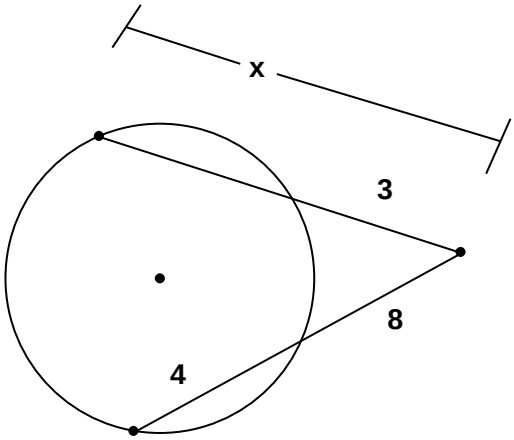
x = _____

2.



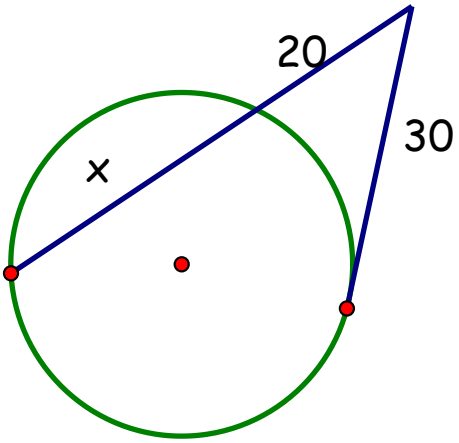
x = _____

3.



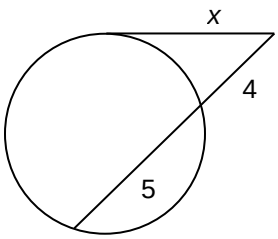
x = _____

4.



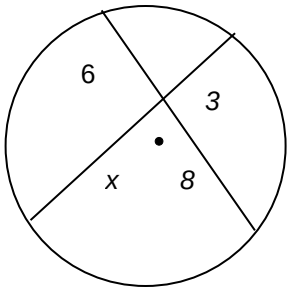
x = _____

5.



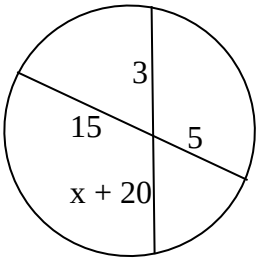
x = _____

6.



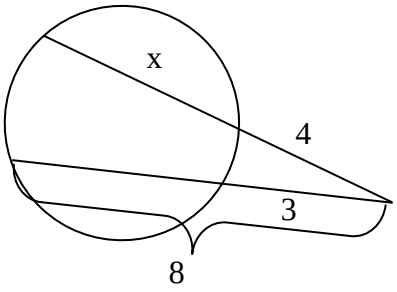
x = _____

7.



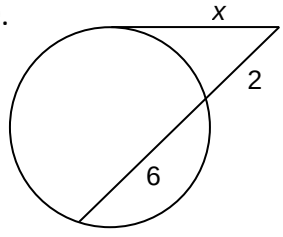
x = _____

8.



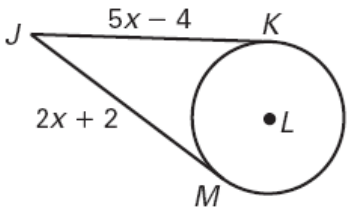
x = _____

9.



x = _____

10.



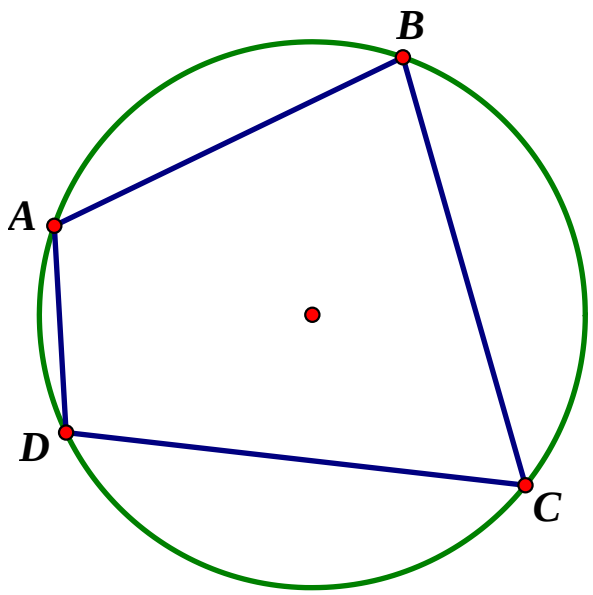
x = _____

Learning Task: Properties of Angles for a Quadrilateral Inscribed in a Circle

Recall: Define a quadrilateral.

Now, you will investigate the relationships among the angles of the quadrilateral inscribed in a circle.

The following quadrilateral, ABCD, is inscribed in a circle.

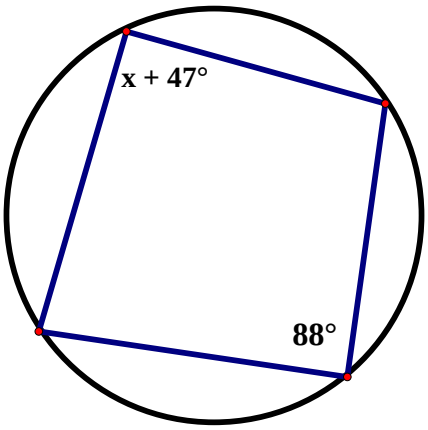


1. What arc does $\angle D$ intercept?
2. Highlight this arc using a colored pencil.
3. What arc does $\angle B$ intercept?
4. Highlight this arc using a colored pencil.
5. What is the sum of the two highlighted arcs?
6. The measure of an inscribed angle is half the measure of its intercepted arc; therefore the $m\angle D + m\angle B =$ _____ .
7. What arc does $\angle A$ intercept?
8. Highlight this arc using a colored pencil.
9. What arc does $\angle C$ intercept?
10. Highlight this arc using a colored pencil.
11. What is the sum of the two highlighted arcs?
12. The measure of an inscribed angle is half the measure of it's intercepted arc; therefore the $m\angle A + m\angle C =$ _____
13. Since $\angle D$ & $\angle B$ and $\angle A$ & $\angle C$ are opposite angles of a quadrilateral inscribed in a circle we can conjecture that:

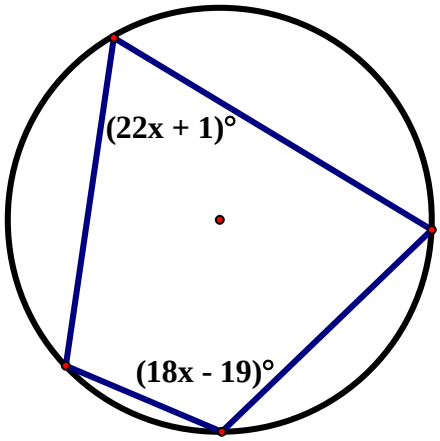
Opposite angles of a quadrilateral inscribed in a circle are _____.

Guided Practice: Find the value of the indicated variable in the inscribed quadrilateral.

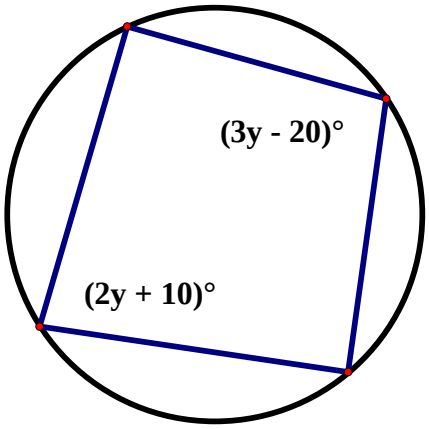
1.



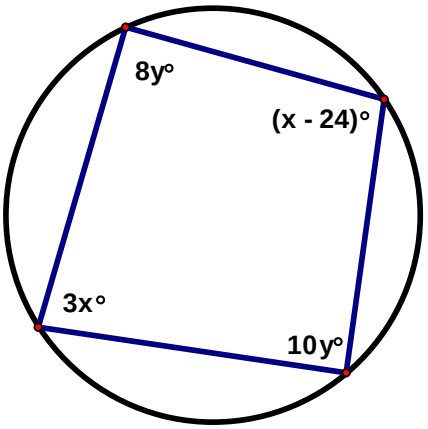
2.



3.

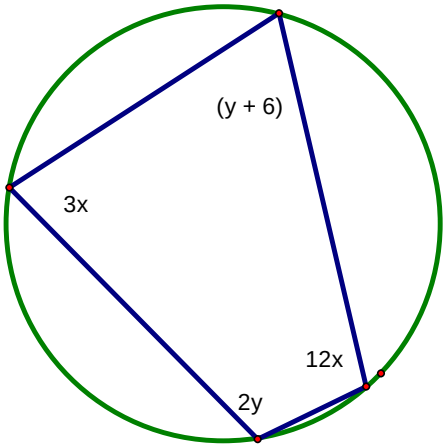


4.



Skills Practice

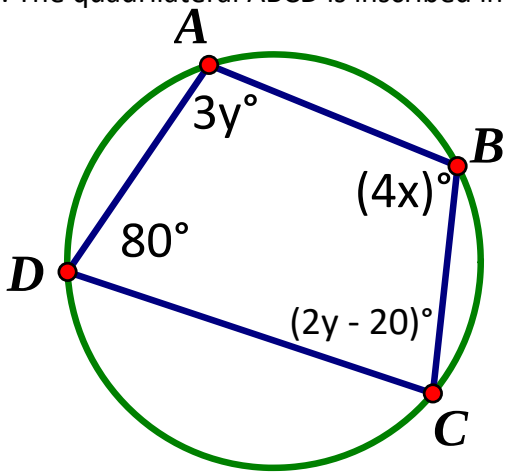
1. Find the value of x and y of the quadrilateral inscribed in the circle.



x = _____

y = _____

2. The quadrilateral ABCD is inscribed in the circle. Solve for the value of x and y.



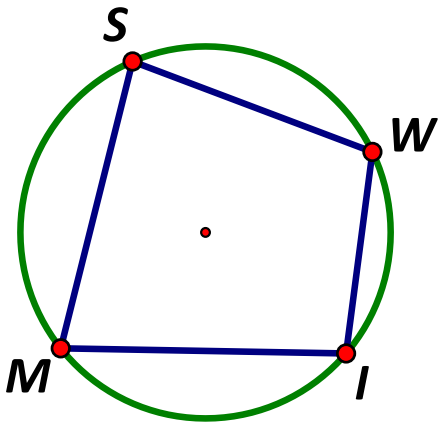
x = _____

y = _____

Also, find angles A, B, C, and D in the quadrilateral shown above.

∠A = _____ ∠B = _____ ∠C = _____ ∠D = _____

3. Given $\angle MSW = (8x+15)^\circ$ *and* $\angle MIW = (7x+15)^\circ$, find the value of x inscribed in the quadrilateral.



x = _____

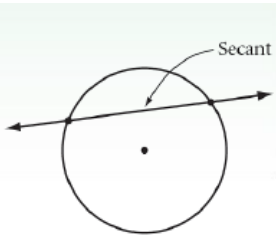
Then, find the measure of $\angle$ MSW and $\angle$ MIW.

$\angle$ MSW = _____

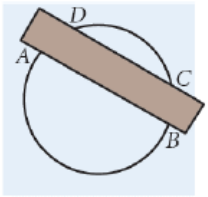
$\angle$ MIW = _____

Arcs by Parallel Lines Investigation

Next, you will investigate arcs formed by parallel lines that intersect a circle. A line that intersects a circle in two points called a secant. A secant contains a chord of the circle and passes through the interior of a circle, while a tangent line does not. Note that a secant is a line while a chord is a segment.



Step 1- On a piece of patty paper, construct a large circle. Lay your straightedge across the circle so its parallel edges pass through the circle. Draw secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ along both edges of the straightedge.



Step 2- Fold your patty paper to compare AD and BC. What can you say about AD and BC?

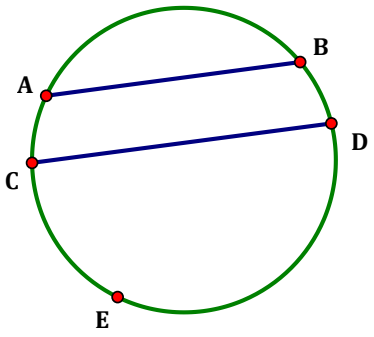
Step 3- Repeat Steps 1 and 2, using either lines paper or another object with parallel edges to construct different parallel secants. Share your results with your partner. Then complete the conjecture below.

Conjecture:
Parallel lines intercept _____ arcs on a circle.

Guided Practice

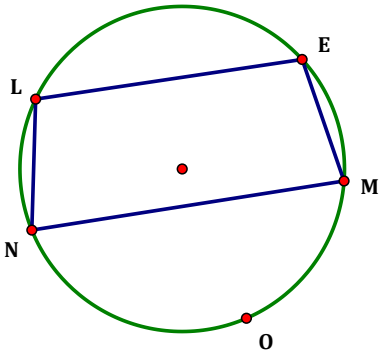
1. $\overline{AB} \| \overline{CD}$, $mAB = 72°$, and $mAC = 25°$.

   Find $mCED$.



2. $\overline{LE} \| \overline{NM}$, $mLE = 86°$, and $mMON = 186°$.

   Find $m\angle LEM$.



3. $\overline{AB} \| \overline{CD}$. Solve for p and q.